MT308 Numerical Analysis

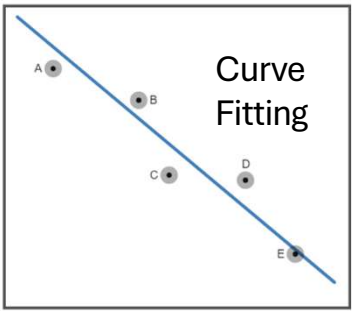
Girne American University
Faculty of Engineering
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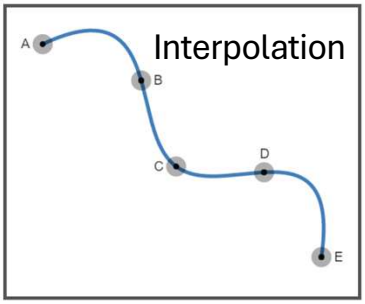
1

Interpolation and curve fitting

- Given a set of data, we might want to approximate the value in between, after, or before what is given. To do this, we can generate a curve by using either **interpolation** or **curve fitting**.
- **Interpolation** is a method of estimating the value of a function at a given point within the range of a set of known data points. It involves finding a polynomial *that* ***fits a set of data points exactly***, rather than just approximating them.
- **Curve fitting**, on the other hand, is the process of finding the best-fitting curve, where the goal is to find a model that captures the *underlying* ***trends in the data***, rather than fitting the data points exactly.

2





Curve Fitting is used for:

- Trend analysis. Predicting values of dependent variable, may include extrapolation beyond data points or interpolation between data points.
- Hypothesis testing. Comparing existing mathematical model with measured data.

Interpolation is used for:

- Estimate missing data points between known values.
- Create smooth curves or functions from discrete data sets.
- Resizing images in image processing
- Animating motion by calculating intermediate frames

3

Curve Fitting

- Curve fitting is the process of finding a mathematical function that best matches a set of data points.
- The curve-fitting process starts with observed data — often noisy or imperfect — and selects a curve (a line, a polynomial, an exponential function, etc.) that approximates the underlying relationship.
- That equation can be used to,
  - ❑Predict future values,
  - ❑Smooth noisy data,
  - ❑Reveal trends,
  - ❑Help analyse systems.

4

Curve Fitting vs Interpolation

	Curve Fitting	Interpolation
Goal	Find a curve that *approximates* the data	Find a curve that passes *exactly* through every data point
Data handling	Ignores noise, finds the best overall trend	Considers every data point, even if it is noisy or unreliable.
Typical use	Real-world noisy measurements	Clean, trusted data tables
Example	Fitting a smooth trend through sensor readings	Finding a value between entries in a math table

5

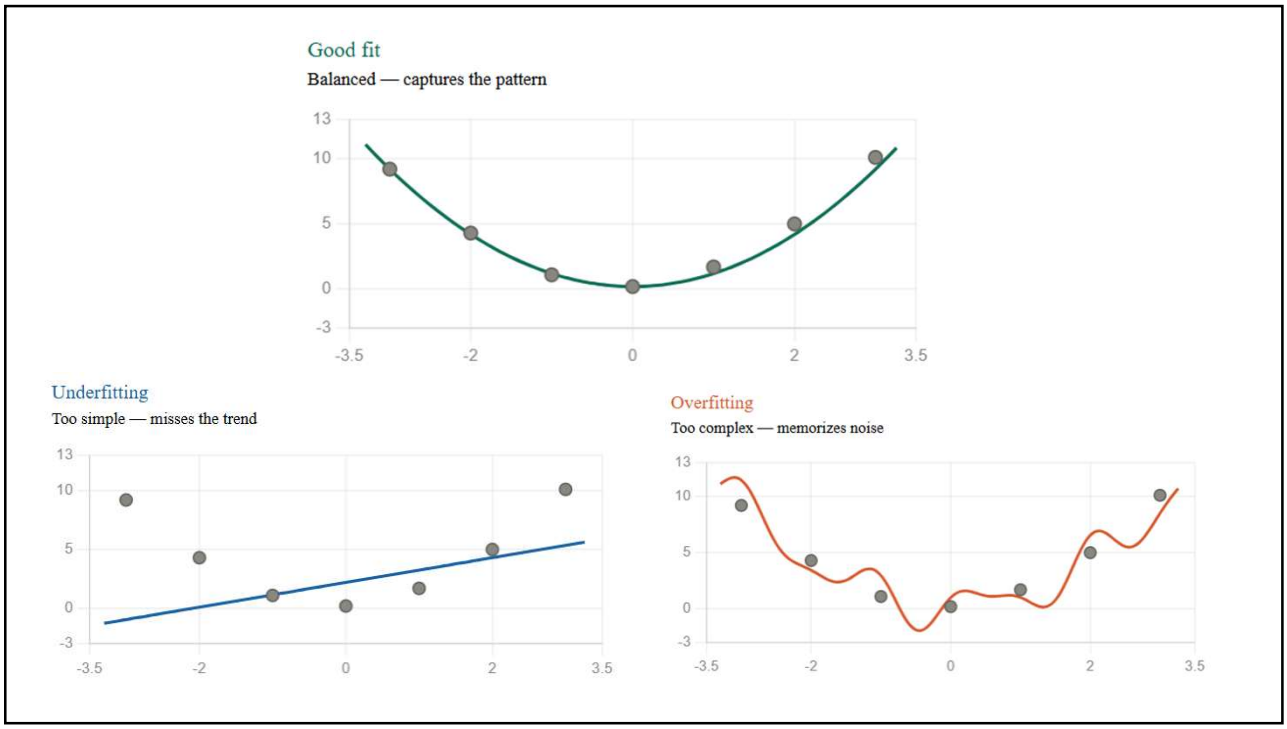
Curve fitting is the process of finding an equation that best represents observed data for analysis and prediction.

Curve fitting is robust to noise and measurement errors.

But it must be used carefully to avoid,

- **Overfitting** occurs when a model learns the data too well, including its noise and random fluctuations.
- In case of overfitting, the training error is very low, but the test/validation error is high.
- **Underfitting** occurs when a model is too simple to learn the true structure of the data.
- In case of underfitting, both the training error and the test/validation errors are very high

6



7

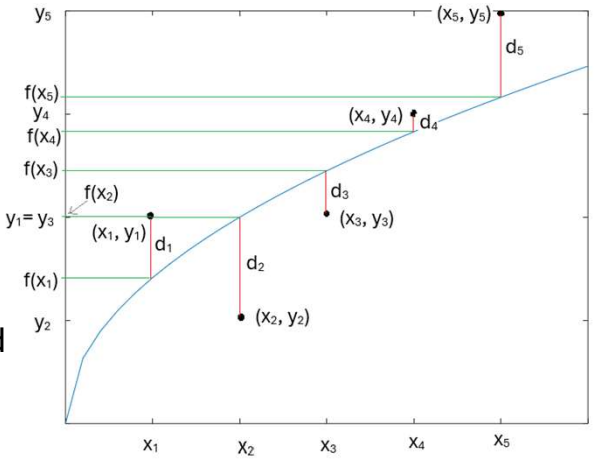
The Least Squares Method

The Least squares is the most common technique used in **curve fitting** and **regression**.

Based on finding the curve that minimizes the sum of squared vertical distances between the data points and the curve.

The squared differences is used instead of raw errors to avoid making a bad fit look perfect by cancelling out the positive and negative errors.

Using squared differences also penalizes large errors more heavily than small ones



$$Error = (d_1)^2 + (d_2)^2 + (d_3)^2 + (d_d)^2 + (d_5)^2$$

$$Error = \sum (y_n - f(x_n))^2 = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + (y_3 - f(x_3))^2 + (y_d - f(x_4))^2 + (y_5 - f(x_5))^2$$

8

Common Types of Curve Fitting

Linear fitting: Fits a straight line.

Example: Distance vs time at constant speed

Polynomial fitting: Fits curved shapes using powers of x.

Example: A GPS system uses polynomial fitting to predict the position

Exponential fitting: For rapid growth or decay

Example: Population growth

Logistic/Sigmoid fitting: For systems that experience growth followed by stabilisation.

Example: Market saturation models

9

Linear Fitting: The least squares line

Least squares fitting is a method that finds the best-fit line by minimizing the sum of the squared differences between data points and the line. $y = ax + b$.

$$S(m,b) = Error = \sum_{i=1}^{n} (y_i - (mx_i + b))^2$$

The variables here are b and b. At the point where the Error function, $S(m,b)$, is minimized, $\frac{\partial S(m,b)}{\partial m} = 0$ and $\frac{\partial S(m,b)}{\partial b} = 0$.

$$m = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{\sum y_i - m\sum x_i}{n}$$

- n = number of data points
- x_i, y_i = observed data values
- m = slope of the best-fit line
- b = y-intercept

10

Example: Fit a least squares line using the data points (-3,7), (0,4), (4,2), (7,-2) and (9,1).

Solution: $\sum x_i = (-3) + 0 + 4 + 7 + 9 = 17$

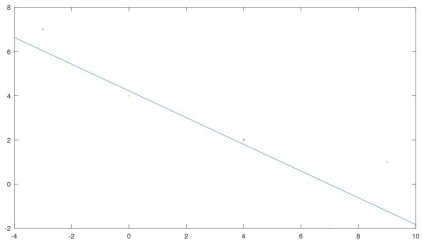
$\sum y_i = 7 + 4 + 2 + (-2) + 1 = 12$

$\sum {x_i}^2 = 9 + 0 + 16 + 49 + 81 = 155$

$\sum x_i y_i = (-3) \cdot 7 + 0 \cdot 4 + 4 \cdot 2 + 7 \cdot (-2) + 9 \cdot 1 = -18$

Then $m = \frac{5\times(-18)-17\times12}{5\times155-155} = -\frac{49}{81} \cong -0.605$ and $b = \frac{12 \ \ (-\frac{49}{81})\times15}{5} \cong 4.215$

So, $y = -0.605x + 4.215$.



11

Assignment 10

Write a MATLAB function that accepts two vectors, x a nd y, which are the abscissa and ordinate values of the points and then uses the least squares method to fit a line (y=mx+b) on the given data points.

12