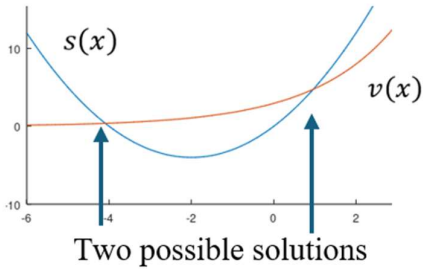
G.A.U. Faculty Of Engineering
MT308/MT208 Numerical Analysis
Spring 2026/ Midterm Exam Questions-Solutions Exam Date: 03 April 2026

1. a) Develop a fixed-point iteration to find the root of $f(x) = x^2 + 4x - 8e^{\frac{x-2}{2}}$.
b) Compute the root up to the fourth iteration.
c) What is the absolute error at the fourth iteration?

Solution: This solution is given in detail to help students better understand how to apply the fixed-point iteration. It was enough to use any part that yielded a correct answer on the exam.

(a) Step 1: Try to guess the position(s) of the possible root(s). The f(x) function may be considered as the difference of the functions $s(x) = x^2 + 4x$ and $v(x) = 8e^{\frac{x-2}{2}}$. The function s(x) is an upward parabola that intersects the x-axis at x=0 and x=-4 and has its minimum at the point (-2,-2). The v(x) is a typical exponential function that intersects the y axis at x=8/e=2.943, where e=exp(1) = e^1=2.71828… is an irrational number. The v(x) does not intersect the x-axis. There are two roots of f(x): The first root is less than -4, and the secondroot is around 1.



Two possible solutions

Step 2: Find an appropriate g(x) function for the iteration $x_{n+1} = g(x_n)$. There are several different possibilities that may be tested:

i. $x = g(x) = \frac{8e^{\frac{x-2}{2}} - x^2}{4}$ and $g'(x) = e^{\frac{x-2}{2}} - \frac{x}{2}$ where $|g'(-4)| = \left|e^{\frac{-4-2}{2}} - \frac{-4}{2}\right| = 2.05$, and $|g'(1)| = \left|e^{\frac{1-2}{2}} - \frac{1}{2}\right| = 0.107$. These results show that this g(x) function is not appropriate to find the first root, which is expected to be less than -4, but it may be used to find the second one. $x_{n+1} = g(x_n) = 2e^{\frac{x_n-2}{2}} - 0.25x_n^2$

ii. $x = g(x) = \pm\sqrt{8e^{\frac{x-2}{2}} - 4x}$ and $g'(x) = \pm\frac{e^{\frac{x-2}{2}} - 1}{\sqrt{2e^{\frac{x-2}{2}} - x}}$ where $g'(-4) = \pm 0.469$, and $g'(1) = 0.853$. Since $|g'(x)| < 1$ around both roots, both iterations converge to the roots:

iii. Let $e^{\frac{x-2}{2}} = (x^2 + 4x)/8$ and then $x = g(x) = 2ln\left(\frac{x^2+4x}{8}\right) + 2$ and its derivative $g'(x) = \frac{4x+8}{x^2+4x}$ where $g'(-4) \to \infty$ and $g'(1) = 2.4$. These results show that the last one cannot be used to find any root

iv. Let $f(x) = x(x+4) - 8e^{\frac{x-2}{2}}$ then $x = g(x) = \frac{8e^{\frac{x-2}{2}}}{x+4}$ where $g'(x) = \frac{4(x+2)e^{\frac{x-2}{2}}}{(x+4)^2}$. Since $g'(-4) = \infty$, and $g'(1) = 0.29$, The g(x) can be used to find only one of the roots.

(b) (c) i. Use the first g(x) function $x_{n+1} = g(x_n) = 2e^{\frac{x_n-2}{2}} - 0.25x_n^2$ with 3 different initial guess values:

n	0	1	2	3	4	$\lvert x_4-x_3\rvert$
x_n	0	0.736	0.928	0.955	0.958	**3.22E-03**
	-2	-0.729	0.378	0.853	0.945	**9.21E-02**
	1	0.963	0.959	0.959	0.958	**5.34E-05**

ii. Use $x_{n+1} = g(x_n) = \pm\sqrt{8e^{\frac{x_n-2}{2}} - 4x_n}$ to find the negative and positive roots using 3 different initial guess values. But remember that the initial guess also must satisfy the condition of $|g'(x_0)| < 1$. You may recognize that $x_0 < 2$ should result in convergent iterations:

n	Positive root: xn			
0	-4	-2	0	1
1	4.049	3.014	1.716	0.923
2	2.468	1.107	0.278	0.988
3	0.487	0.831	1.507	0.933
4	1.344	1.065	0.474	0.980

Negative root: xn			
-4	-2	0	1
-4.049	-3.014	-1.716	-0.923
-4.073	-3.565	-2.848	-2.355
-4.083	-3.841	-3.479	-3.214
-4.089	-3.974	-3.799	-3.667

$\lvert x_4-x_3\rvert$	0.856	0.234	1.033	0.047	0.005	0.133	0.320	0.453

Note: Finding only one value was enough to answer this question.

iv. Use $x_{n+1} = g(x_n) = \frac{8e^{\frac{x_n-2}{2}}}{x_n+4}$ to find the root around x=1. Use different initial guess values that satisfy $|g'(x_0)|<1$:

n	0	1	2	3	4
x_n	0	0.736	0.898	0.941	0.954
	1	0.970	0.962	0.959	0.959
	2	1.333	1.075	0.993	0.968

$\lvert x_4-x_3\rvert$
1.22E-02
7.02E-04
2.43E-02

2. (a) Please use the Bisection method to find the root of $f(x) = x^2 + 4x - 8e^{\frac{x-2}{2}}$ with the initial interval -3<x<4 Continue five iterations.
(b) Calculate the relative error of this calculation at the fifth iteration.

a)

	a	c=(a+b)/2	b	f(a)	f(c)
0	-3.000	0.500	4.000	-3.66	-1.53
1	0.500	2.250	4.000	-1.53	5.00
2	0.500	1.375	2.250	-1.53	1.54
3	0.500	0.938	1.375	-1.53	-0.07
4	0.938	1.156	1.375	-0.07	0.72
5	0.938	1.047	1.156	-0.07	0.32

b) Absolute Error: 0.109
Relative Error: 10.45%

3. **Write a MATLAB script to find the relative error at the end of the 15th iteration when question 2 is solved by using the Regula Falsi method.**

Solution:

```
f=@(x) x^2+4*x-8*exp((x-2)/2);
a=-3;
b=4;
c=(f(b)*a-f(a)*b)/(f(b)-f(a));
for it=1:15
  c0=c; %Required for error calculation
  c=(f(b)*a-f(a)*b)/(f(b)-f(a));
  if f(a)*f(c)<0,
    b=c;
  else
    a=c;
  end
end
fprintf("The value of root is %f\nThe relative error is %f",c,abs((c-c0)/c))
```

4. **(a) Please use the Newton-Raphson method to find the root of $f(x)=12x^2+13x-14$ with two initial guesses (i) $x_0=-1$ and (ii) $x_0=3$. Continue 4 iterations.**
   (b) Calculate the relative error of this calculation at the 4th iteration.

Solution: Since $f'(x) = 24x + 13$ then $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{12\ x_n^2+12x_n-1}{24\ x_n+13}$

n	0	1	2	3	4
x_n	-1	-2.500	-1.880	-1.752	-1.750
x_n	3	1.453	0.835	0.682	0.667

$[x_4 - x_3]$	Relative Error $\left[\frac{x_4 - x_3}{x_4}\right]$
0.002	0.14%
0.014	2.15%

5. **Write a MATLAB script that asks the user to enter a square matrix (A) and a vector (v), where the length is at least 4. The script must verify that A is a square matrix, and that its size matches the length of v. Then, (a) Construct the augmented matrix B; (b) Multiply the first row of B by 3 and multiply the 3rd row of B by -2; (c) replace the first half of the second row of B with ones; (d) replace the second half of the fourth row of B with zeros.**

Solution:

```
A=input("Enter a matrix at least size of 4");
v=input("Enter a column vector in the same size of the matrix you entered");
[n,k]=size(A);
[s,t]=size(v);
if n==k,
  if n==s
    B=[A, v];
  elseif n==t,
    B=[A, v'];
  else
    disp("The size of v does not match the size of A");
  end
else
  disp("A must be a square matrix");
end
B(1,:)=3*B(1,:);
B(3,:)=-2*B(3,:);
B(2,1:n/2)=ones(1,n/2);
B(4,1+n/2:end)=zeros(size(B(4,n/2:end)));
```

6. **Please use the Gauss Elimination method to solve the following linear system:**

$$\begin{aligned} -3x+y-5z&=1\\ x-3y-2z&=10\\ -3x+y+4z&=-8 \end{aligned}$$

Solution: The augmented matrix: $\left[\begin{array}{ccc|c} -3 & 1 & -5 & 1 \\ 1 & -3 & -2 & 10 \\ -3 & 1 & 4 & -8 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -2 & 10 \\ -3 & 1 & -5 & 1 \\ -3 & 1 & 4 & -8 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 10 \\ 0 & -8 & -11 & 31 \\ 0 & -8 & -2 & 22 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -2 & 10 \\ 0 & -8 & -11 & 31 \\ 0 & -8 & -2 & 22 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -2 & 10 \\ 0 & -8 & -11 & 31 \\ 0 & 0 & 9 & -9 \end{array}\right]$$

Then z=-9/9=-1, y=(31+11×(-1))/(-8)=-2.5 and x=10+3y+2z=0.5

7. **Write a MATLAB function (Use your first name as the name of the function) that accepts 3 input arguments a, b, and c and returns x_1 and x_2, which are the roots of the function $f(x) = ax^2 + bx + c$ using the expression $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.**

Solution:

```
function [x1, x2] = std_name (a,b,c)
    delta=b*b-4*a*c;
    x1=(-b-sqrt(delta))/2/a;
    x2=(-b+sqrt(delta))/2/a;
end
```