G.A.U. Faculty Of Engineering And Architecture
MT308 Numerical Analysis

Duration: 90 Minutes | Midterm Examination | 16th of July. 2025

Dr. Kamil Dimililer

Solutions

Notes: Use rounded numbers <u>at least 3 decimal digits</u>.

1. a) Develop a fixed-point iteration to find the root of

$$f(x) = e^{-(x-1)/4} - \frac{x}{3}$$

b) Compute the root up to fourth iteration.

c) What is the absolute error at the fourth iteration?

Solution: if $f(x) = e^{-(x-1)/4} - \frac{x}{3} = 0$ then $x = g(x) = 3e^{-(x-1)/4}$ where $g'(x) = -\frac{3}{4}e^{-(x-1)/4}$ and $|g'(x)| = \frac{3}{4}e^{-(x-1)/4} < 1$ if x>-0.15. therefore this iteration converges for any positive root by the following iteration formula:

$$x_{n+1} = g(x_n) = 3e^{-(x_n-1)/4}$$

The following tables show iterations for x_0=0.5, x_0=1, x_0=2 and x_0=6

n	x_n	|f(x)|	$\|x_{n+1}-x_n\|$		x_n	|f(x)|	$\|x_{n+1}-x_n\|$
0	0.5	0.966482			1	0.666667	
1	3.399445	0.584261	0.382221		3	0.393469	0.2732
2	1.646663	0.301838	0.282423		1.819592	0.2082	0.18527
3	2.552176	0.172343	0.129495		2.444191	0.117785	0.09042
4	2.035147	0.093605	0.078738		2.090837	0.064368	0.05342

n	xn	|f(x)|	|xn+1-xn|		xn	|f(x)|	|xn+1-xn|
0	2	0.112134			6	1.713495	
1	2.336402	0.062819	0.049315		0.859514	0.749241	0.964255
2	2.147945	0.03454	0.028279		3.107236	0.445259	0.303981
3	2.251566	0.019193	0.015347		1.771458	0.234107	0.211152
4	2.193988	0.010603	0.008589		2.473781	0.132784	0.101323

2. Write a code to solve Question 1.

```
function r=solve_q1(x)
    g=@(x) 3*exp(-(x-1)/4);
    for i=1:4,
      r=g(x)
      x=r;
    end
end
```

```
>> solve_q1(2)
r = 2.3364
r = 2.1479
r = 2.2516
r = 2.1940
ans = 2.1940
```

```
function r=solve_q1(x,err)
    f=@(x) exp(-(x-1)/4)-x/3;
    g=@(x) 3*exp(-(x-1)/4);
    r=g(x);
    while abs(f(x))>err,
      r=g(x)
      x=r;
    end
end
```

3. (a) Please use the Newton Raphson method to find the root of $f(x)=2x^2+6x-54$ with an initial point $x_0=2$ and continue 5 iterations.,
(b) Calculate the relative error of this calculation at the fifth iteration.

Solution: $x_{n+1} = \frac{2x_n^2+6x_n-54}{4x_n+6}$

n	x_n
0	2
1	4.428571
2	3.931153
3	3.908375
4	3.908327
5	3.908327

Exact roots: 3.9083269 and -6.9083269

Relative error≈0%

4. (a) Please use the Regula-Falsi method to find the root of $f(x)=2\ln(2x-3)-x/2$ with the initial interval $2 <x<4$. Continue five iterations.
(b) Calculate the relative error of this calculation at the fifth iteration.
Solution:

iter.	a	c	b	f(a)	f(c)	f(b)
0	2	2.901357	4	-1	0.610498	1.218876
1	2	2.559676	2.901357	-1	0.222383	0.610498
2	2	2.457857	2.559676	-1	0.071252	0.222383
3	2	2.427403	2.457857	-1	0.021859	0.071252
4	2	2.41826	2.427403	-1	0.006616	0.021859
5	2	2.415512	2.41826	-1	0.001994	0.006616

Relative Error: $|(c_5-c_4)/c_5|=0.03\%$

```
#MT308 Summer 2025- midterm Question 5
f=@(x) 2*x^2+6*x-54;

a=input('Enter the lower bound: ')
b=input('Enter the upper bound: ')
if f(a)*f(b)>0,
  disp('Incorrect bounds')
elseif f(a)==0
  c=a;
elseif f(b)==0
  c=b;
else
  c=(a+b)/2;
  while abs(f(c))>1E-5,
     c=(a+b)/2;
     if f(c)*f(a)>0
       a=c;
     else
       b=c;
     end
  end
end
printf('the root is %.3f\n',c)
```

5. Write a code to solve the Question 3 by using the bisection method so that the absolute error is less than 1E-5.

Output:

```
Enter the lower bound: 1
a = 1
Enter the upper bound: 9
b = 9
the root is 3.908
```

6. Please use Gauss Elimination method to solve the following linear system:

$$3x-2y+z=3$$
$$2x+2y-5z=-1$$
$$-x-y+5z=1$$

Solution: The augmented matrix:

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 3 \\ 2 & 2 & -5 & -1 \\ -1 & -1 & 5 & 1 \end{array}\right]$$

You may follow different steps. But since row 2 and row 3 has a potential to eliminate two columns at one step, reorganizing the rows may reduce the number of calculations (optional):

Therefore,

Multiply the new first row by 3 and add it to the second one. Multiply the first row by 2 and add it to the third one.

$$\left[\begin{array}{ccc|cc} -1 & -1 & 5 & 1 & \text{(was Row 3)} \\ 3 & -2 & 1 & 3 & \text{(was Row 1)} \\ 2 & 2 & -5 & -1 & \text{(was Row 2)} \end{array}\right]$$

Use back substitution to find

$$\left[\begin{array}{ccc|c} -1 & -1 & 5 & 1 \\ 0 & -5 & 16 & 6 \\ 0 & 0 & 5 & 1 \end{array}\right]$$

$$\boxed{x=\frac{14}{25},\quad y=-\frac{14}{25},\quad z=\frac{1}{5}}$$

7. Suppose that you have a system (computer or calculator) that stores real number in 12 bits. Mantissa of a real number is stored in 6 bits, and the exponent is stored in 5 bits and one bit is used to represent the sign of the real number. The 2's complement method is used to represent negative exponent values.
   (a) Please calculate the relative error of 2.3+1.1
   (b) (i) Find the smallest positive number.
   (ii) What is the highest number?
   (iii) What is the lowest number?

that can be represented by this system.

Solution: Remember real numbers stored in the form $1.xxx\ldots\times 2^n$ for example $0.5_D = 1.0_B * 2^{-1}$

a) $2.3_{DEC} \approx 10.0100110011..._{BIN}$ then $2.3=1.0010011...\times 2^1$, Mantissa:001001 and exponent: 00001. In this system $2.3\approx 1.001001\times 2^1=73/32=2.28125$
$1.1_{DEC} \approx 1.000110011..._{BIN}$ then $1.1=1.000110011...\times 2^0$, where the mantissa is 000110 and exponent is 00000. Then 1.1 is represented as $1.000110\times 2^0=1.09375$
This system will calculate $2.3+1.1\approx 2.28125+1.09375=3.375$ instead of 2.3+1.1=3.4.
Therefore, the relative error is (3.4-3.375)/3.4=0.735%

b) (i) The smallest positive number: $+1.000000\times 2^N$, where the exponent, N has its minimum value: $10000_{bin,signed}=-10000_{bin}=-16_{dec}$. Therefore the minimum positive number is $1.000000\times 2^{-16}=1.525877\times 10^{-5}$.
(ii) The highest number is $1.111111\times 2^{15}=65024_{dec}$
(iii) The lowest number is $-1.111111\times 2^{15}=-65024_{dec}$